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Abstract

Let $\mathcal{G}$ denote a split simply connected almost simple p-adic group. We study the unramified C^*-algebra of $\mathcal{G}$ and prove that the rank of the K-theory group K_0 is the connection index $f(\mathcal{G})$.

1 Introduction

Let k be a global field, let $\mathbb{A}_k$ denote ring of adeles over k. Let π denote an automorphic representation of the adelic group $G(\mathbb{A}_k)$. Then we have the restricted product

$$\pi = \otimes_v \pi_v$$

over all the places v of k. All but finitely many of the local representations π_v admit K_v-fixed vectors: they are *unramified*. In this Note, we investigate the unramified C^*-algebra attached to a local group G_v.

One point of interest is that one has to go outside the affine Weyl group W_a to the extended affine Weyl group W_a': in fact, the crucial finite group in this context is the quotient group W_a'/W_a.

Quite specifically, let F be a local nonarchimedean field of characteristic 0, let $\mathcal{G}$ be the group of F-rational points in a split, almost simple, simply connected, semisimple linear algebraic group defined over F, for example $\mathrm{SL}(n)$. Let $\mathcal{S}$ denote a maximal split torus of $\mathcal{G}$. Let $\mathcal{G}^\vee$, $\mathcal{S}^\vee$ denote the Langlands dual groups, and let G, T denote maximal compact subgroups:

$$G \subset \mathcal{G}^\vee, \quad T \subset \mathcal{S}^\vee.$$

Then G is a compact Lie group with maximal torus T.

We will consider the unramified C^*-algebra $\mathfrak{A} = \mathfrak{A}(\mathcal{G}) = C^*_{nr}(\mathcal{G})$ of $\mathcal{G}$. Let

$$\pi(t) := \mathrm{Ind}_{\mathcal{SU}}^{\mathcal{G}}(t \otimes 1)$$

be the unramified unitary principal series of $\mathcal{G}$, so that $t \in \Psi(\mathcal{S})$ is an unramified unitary character of $\mathcal{S}$. Let $\mathcal{C}(t)$ be the commuting algebra of $\pi(t)$. Then

$$\mathcal{C}(t) \cong \mathbb{C}[R(t)],$$

where $R(t)$ is the R-group of t, see [6]. The R-groups are tabulated by Keys [6].

Let $\Phi^\vee$ be the coroot system associated to Φ. Let $Q(\Phi)$ be the root lattice, $P(\Phi)$ be the weight lattice; each weight takes integer values on the coroot lattice. Let $P(\Phi)/Q(\Phi)$ be the quotient group. The order of the group $P(\Phi)/Q(\Phi)$ is called the *connection index* $f = f(\mathcal{G})$ of $\mathcal{G}$.

The abelian groups P/Q are tabulated in [4, Plates I–X, p.265–292]. For $A_n, B_n, C_n, D_n, E_6, E_7, E_8, F_4, G_2$ they are, respectively

$$\mathbb{Z}/(n+1)\mathbb{Z}, \quad \mathbb{Z}/2\mathbb{Z}, \quad \mathbb{Z}/2\mathbb{Z}, \quad \mathbb{Z}/4\mathbb{Z}\ (n\ odd), \quad \mathbb{Z}/2\mathbb{Z}\times\mathbb{Z}/2\mathbb{Z}\ (n\ even)$$

$$\mathbb{Z}/3\mathbb{Z}, \quad \mathbb{Z}/2\mathbb{Z}, \quad 0, \quad 0, \quad 0$$

Here is the striking fact: *the maximal R-groups are isomorphic to the fundamental groups P/Q*. To see this, it is enough to compare the R-group computations in [6] with the above list. In this Note, we attempt to explain this fact, via the elementary geometry of the Lie algebra $\mathfrak{t}$: the vector space $\mathfrak{t}$ has the structure of a Euclidean apace tessellated with alcoves. In the present context, we need a refinement of this tessellation. For example, if $\mathcal{G} = \mathrm{SL}(3)$, then the standard tiling of the Euclidean plane by equilateral triangles is refined to produce a tiling by isosceles triangles (see the Figures in section 3). Reducibility in the unramified unitary principal series is determined by the fundamental group P/Q.

Theorem 1.1. *Let $\mathcal{G}$ be simply connected, let $\mathfrak{A} = C^*_{nr}(\mathcal{G})$ and let $f(\mathcal{G})$ be the connection index of $\mathcal{G}$. Then we have*

$$\begin{aligned} K_0(\mathfrak{A}) &= \mathbb{Z}\oplus\mathbb{Z}\oplus\cdots\oplus\mathbb{Z} \qquad (f(\mathcal{G})\ \textit{copies of}\ \mathbb{Z}) \\ K_1(\mathfrak{A}) &= 0 \end{aligned}$$

K_0 detects the L-packet in the unramified unitary principal series with the maximal number of constituents.

2 The alcoves in $\mathfrak{t}$

Our reference at this point is [4, Lie IX, p. 309–327]. The proof hinges on the distinction between the affine Weyl group and the extended affine Weyl group. Let $\mathfrak{t}$ denote the Lie algebra of T, and let $\exp : \mathfrak{t} \to T$ denote the exponential map. The kernel of exp is denoted $\Gamma(T)$. Denote by $N(G,T)$ the subgroup of $\Gamma(T)$ generated by the nodal vectors. The quotient $\Gamma(T)/N(G,T)$ can be identified with the fundamental group of G. Thanks to the identification of the nodal vector K_α with the coroot $\alpha^\vee$, the group

$N(G,T)$ can be identified with the group $Q(\Phi^\vee)$ generated by the set $\Phi^\vee$ of coroots.

The affine Weyl group is $W_a = N(G,T) \rtimes W$ and the extended affine Weyl group is $W'_a = \Gamma(T) \rtimes W$; the subgroup W_a of W'_a is normal. The group W (resp. W_a) operates simply-transitively on the set of chambers (resp. alcoves). Let C be a chamber and A an alcove. Then $\overline{C}$ (resp. $\overline{A}$) is a fundamental domain for the operation of W on $\mathfrak{t}$ (resp. of W_a on $\mathfrak{t}$). Let H_A be the stabilizer of A in W'_a. Then W'_a is the semi-direct product

$$W'_a = W_a \rtimes H_A.$$

We have to consider the C^*-summand, $\mathfrak{A}(G)$ in the reduced C^*-algebra $C^*_r(G)$ which corresponds to the unramified unitary principal series, see [6]. The algebra $C^*_r(G)$ is defined as follows. We choose a left-invariant Haar measure on G, and form a Hilbert space $L^2(G)$. The left regular representation λ of $L^1(G)$ on $L^2(G)$ is given by

$$(\lambda(f))(h) = f * h$$

where $f \in L^1(G)$, $h \in L^2(G)$ and $*$ denotes the convolution. The C^*-algebra generated by the image of λ is the reduced C^*-algebra $C^*_r(G)$. Let $\mathfrak{K}$ denote the C^*-algebra of compact operators on the standard Hilbert space H.

At this point we use *Langlands duality.* The Langlands dual of $\mathcal{G}$ is the complex reductive group $\mathcal{G}^\vee$ with maximal torus $S^\vee$. Let G be a maximal compact subgroup of $\mathcal{G}^\vee$, let T be the maximal compact subgroup of $\mathcal{S}^\vee$.

The group $\Psi(\mathcal{S})$ of unramified unitary characters of $\mathcal{S}$ is isomorphic to T. The unramified C^*-algebra $C^*_{nr}(G)$ is given by the fixed point algebra

$$\begin{aligned} C^*_{nr}(G) &= C(T, \mathfrak{K})^W \\ &= \{f \in C(T,\mathfrak{K}) : f(wt) = \mathfrak{c}(w:t) \cdot f(t), w \in W\} \end{aligned}$$

where $\mathfrak{a}(w:t)$ are normalized intertwining operators, and

$$\mathfrak{c}(w:t) = \mathrm{Ad}\, \mathfrak{a}(w:t)$$

as in [7]. Then $\mathfrak{a} : W \to C(T, U(H))$ is a 1-cocycle:

$$\mathfrak{a}(w_2 w_1 : t) = \mathfrak{a}(w_2 : w_1 t)\mathfrak{a}(w_1 : t).$$

We have $\overline{A} \subset \mathfrak{t}$. The group H_A is defined as the stabilizer in W'_a of $\overline{A}$:

$$H_A := \{w \in W'_a : w\overline{A} \subset \overline{A}\}$$

Then H_A is a finite abelian group [4, Ch.IX, p.326]. View $\mathfrak{t}$ as an additive group, and form the Euclidean group $\mathfrak{t} \rtimes O(\mathfrak{t})$. We have $W_a' \subset \mathfrak{t} \rtimes O(\mathfrak{t})$ and so W_a' acts as affine transformations of $\mathfrak{t}$.

The *special points* of W_a are the elements x of $\mathfrak{t}$ such that $\exp x \in C(G)$, see [4, Ch. IX, p.326]. Let S be the set of special points of $\overline{A}$. The group H_A acts freely on S, by [4, Ch. IX, p.327]. Let x_0 denote the barycentre of S, i.e.

$$x_0 := \frac{1}{|S|} \cdot \sum_{s_j \in S} s_j.$$

The group H_A will permute the special points $s \in S$. Note that

$$w \cdot x_0 = \frac{1}{|S|} \sum_{s_j \in S} w \cdot s_j = \frac{1}{|S|} \sum_{s_j \in S} s_j = x_0$$

so that x_0 is H_A-fixed. Then $\overline{A}$ is equivariantly contractible to x_0:

$$r_t(x) := (1-t)x_0 + tx \tag{1}$$

with $0 \leq t \leq 1$. This is an affine H_A-equivariant contraction from $\overline{A}$ to x_0.

Lemma 2.1. *Let $t_0 = \exp x_0$. There is an isomorphism of abelian groups:*

$$H_A \simeq W(t_0) \simeq R(t_0)$$

where $R(t_0)$ is the R-group of t_0.

Proof. The exponential map $\exp : \mathfrak{t} \to T$ commutes with the action of W. Let $h = \gamma w = (\gamma, w) \in \Gamma(T) \rtimes W$. Then

$$hx_0 = x_0 \implies \gamma \in \Gamma(T) \implies \exp hx_0 = \exp wx_0 = w \exp x_0 = wt_0$$

This determines the map

$$H_A \longrightarrow W(t_0), \quad \gamma w \mapsto w$$

Now

$$w(t_0) = t_0 \implies (\exists \gamma \in \Gamma(T))\, \gamma(wx_0) = x_0 \implies \gamma w \in H_A$$

so that $H_A \to W(t_0)$ is surjective. The map is injective because

$$(\gamma_1, w) = (\gamma_2, w) \implies \gamma_1 = \gamma_2$$

This creates the canonical isomorphism of abelian groups:

$$H_A \simeq W(t_0)$$

In fact, $W(t_0)$ is the image of H_A in the canonical map $W'_a \to W$ is $W(t_0)$:

$$\Gamma(T) \rtimes W \to W, \quad H_A \simeq W(t_0)$$

In general, we have $R(t_0) \subset W(t_0)$. The calculations of Keys [6, Theorem in §3] prove that $R(t_0) = W(t_0)$. $\square$

Theorem 2.2. *The group $K_0(C^*_{nr}(G))$ is free abelian on $f(\mathcal{G})$ generators, and $K_1(C^*_{nr}(G)) = 0$.*

Proof. We have the exponential map $\exp : \mathfrak{t} \to T$. We lift f from T to a periodic function F on $\mathfrak{t}$, and lift $\mathfrak{a}$ from a 1-cocycle $\mathfrak{a} : W \to C(T, U(H))$ to a 1-cocycle $\mathfrak{b} : W'_a \to C(\mathfrak{t}, U(H))$:

$$F(x) := f(\exp x), \quad \mathfrak{b}(w' : x) := \mathfrak{a}(w : \exp x)$$

with $w' = (\gamma, w)$. The semidirect product rule is

$$w'_1 w'_2 = (\gamma_1, w_1)(\gamma_2, w_2) = (\gamma_1 w_1(\gamma_2), w_1 w_2).$$

Note that $\mathfrak{b}$ is still a 1-cocycle:

$$\begin{aligned}
\mathfrak{b}(w'_2 w'_1 : x) &= \mathfrak{a}(w_2 w_1 : \exp x) \\
&= \mathfrak{a}(w_2 : w_1(\exp x)\mathfrak{a}(w_1 : \exp x) \\
&= \mathfrak{a}(w_2 : \exp w_1 x)\mathfrak{b}(w'_1 : x) \\
&= \mathfrak{a}(w_2 : \exp \gamma w_1 x)\mathfrak{b}(w'_1 : x) \\
&= \mathfrak{b}(w'_2 : w'_1 x)\mathfrak{b}(w'_1 : x)
\end{aligned}$$

Now we define

$$\mathfrak{d}(w : x) := Ad\, \mathfrak{b}(w : x), \qquad w \in W'_a \tag{2}$$

The fixed algebra $C^*_{nr}(G)$ is as follows:

$$\{F \in C(\mathfrak{t}, \mathfrak{K}) : F(wx) = \mathfrak{d}(w : x) \cdot F(x), w \in W'_a,\ x \in \mathfrak{t},\ F \text{ periodic}\}$$

Now F is determined by its restriction to $\overline{A}$. Upon restriction, we obtain

$$C^*_{nr}(G) \simeq \{f \in C(\overline{A}, \mathfrak{K}) : f(wx) = \mathfrak{d}(w : x) \cdot f(x), w \in H_A,\ x \in \overline{A}\}$$

We want to make the $\mathfrak{b}(w, x)$ independent of $x \in \overline{A}$. At this point, we adapt a proof in Solleveld [9]. We will write $H = H_A$. Let

$$\mathfrak{C} := C(\overline{A}, \mathfrak{K}), \qquad \mathfrak{B} := C(\overline{A}, \mathfrak{L}(V))$$

so that $C^*_{nr}(G) = \mathfrak{C}^H$. Let $\mathfrak{C} \rtimes H$ be the crossed product of $\mathfrak{C}$ and H with respect to the action of H on $\mathfrak{C}$ via the intertwiners $\mathfrak{b}(w : x)$. Let r_t be defined as in Eq.(1). Define

$$p_t(x) := \frac{1}{|H|} \sum_{w \in H} \mathfrak{b}(w : r_t x) \cdot w \tag{3}$$

Then $p_t \in \mathfrak{B} \rtimes H$ is an idempotent by Eq.(3). By [8], we have an isomorphism of C^*-algebras:

$$\phi_1 : \mathfrak{C}^H \simeq p_1(\mathfrak{C} \rtimes H)p_1, \quad \sigma \mapsto p_1 \sigma p_1$$

Clearly the idempotents p_t are all homotopic, so they are conjugate in $\mathfrak{B} \rtimes H$, see [3, Proposition 4.3.3], e.g. $p_1 = a\, p_0\, a^{-1}$. Now $\mathfrak{C} \rtimes H$ is a C^*-ideal in $\mathfrak{B} \rtimes H$, and so we have

$$\begin{aligned} \mathfrak{C}^H &\simeq p_1(\mathfrak{C} \rtimes H)p_1 \\ &= ap_0a^{-1}(\mathfrak{C} \rtimes H)ap_0a^{-1} \\ &= ap_0(\mathfrak{C} \rtimes H)p_0a^{-1} \\ &\simeq p_0(\mathfrak{C} \rtimes H)p_0 \end{aligned}$$

as conjugate C^*-algebras in $\mathfrak{B} \rtimes H$. Define

$$\mathfrak{C}^H(0) := \{f \in C(\overline{A}, \mathfrak{K}) : f(wx) = \mathfrak{d}(w : x_0) \cdot f(x), w \in H_A,\ x \in \overline{A}\}$$

We have the the C^*-algebra isomorphism

$$\mathfrak{C}^H(0) \simeq p_0(\mathfrak{C} \rtimes H)p_0, \quad \sigma \mapsto p_0 \sigma p_0$$

and so

$$\mathfrak{C}^H \simeq p_0(\mathfrak{C} \rtimes H)p_0 \simeq \mathfrak{C}(0)^H$$

We have replaced the intertwiners $\mathfrak{b}(w : x)$ by the *constant* (i.e. independent of x) intertwiners $\mathfrak{b}(w : x_0)$.

To the algebra $\mathfrak{C}(0)^H$ we will apply the homotopy r_t. This shows that $\mathfrak{C}^H(0)$ is homotopy-equivalent to its fiber $\mathfrak{K}(V_0)^H$ over x_0. Each element in $\mathfrak{K}(V_0)^H$ must commute with the intertwiners $\{\mathfrak{b}(x_0 : h) : h \in H_A\}$. Now we have

$$\{\mathfrak{b}(x_0 : h) : h \in H_A\} = \{\mathfrak{a}(t_0 : w) : w \in W(t_0)\}$$

by Lemma 2.1 and Eq.(2). So each element in $\mathfrak{K}(V_0)^H$ must commute with the commuting algebra

$$\mathcal{C}(t_0) \simeq \mathbb{C}[H_A]$$

Since H_A is an abelian group of order f, we infer that

$$\mathfrak{K}(V_0)^H \simeq \mathfrak{K} \oplus \ldots \oplus \mathfrak{K}$$

with f copies of $\mathfrak{K}$. It is now immediate that $K_0(\mathfrak{A}) = \mathbb{Z}^f, \ K_1(\mathfrak{A}) = 0$.

The logic of the above proof is this:

$$C^*_{nr}(G) = \mathfrak{C}^H \simeq \mathfrak{C}^H(0) \sim \mathfrak{K}(V_0)^H \simeq \mathfrak{K}^f$$

□

The unramified unitary principal series is reducible at t_0, and it splits the Hilbert space V_0 into f irreducible subspaces $H_1, \ldots, H_f$. Let $E_j : V_0 \to H_j$ be a rank-one projection. Let $e_j \in \mathfrak{A}$ be such that $e_j(x_0) = E_j$. Then $\{e_1, \ldots, e_f\}$ is a set of generators in $K_0(\mathfrak{A})$.

3 Examples

The exponential map $\exp : \mathfrak{t} \to T$ induces a homeomorphism

$$\overline{A}/H_A \simeq T/W$$

by [4, Ch. IX, p.326], and $\overline{A}/H_A$ is equivariantly contractible. The map r_t in Eq. (1) descends to the quotient $\overline{A}/H_A$ and shows that the quotient $\overline{A}/H_A$ is a contractible space. Therefore, T/W is contractible.

If $\mathcal{G} = E_8, \ F_4$ or G_2 then we have $P/Q = 0$ and so the connection index $f = 1$, see [4, Plates VIII, IX, X]. Therefore

$$T/W \simeq \overline{A}.$$

This is a contractible space. There are no L-packets. The K-theory of the unramified unitary principal series of $\mathcal{G}$ is that of a point.

From now on, let $\mathcal{G} = \mathrm{SL}(\ell)$ with ℓ a prime number. We have $\mathcal{G}^\vee = \mathrm{PGL}(\ell, \mathbb{C}), \ G = \mathrm{PU}(\ell, \mathbb{C})$ and T is a maximal torus in $\mathrm{PU}(\ell, \mathbb{C})$. Then the Weyl group W is the symmetric group S_ℓ. Let γ denote the standard ℓ-cycle $(123 \ldots \ell) \in S_\ell$. The fixed set of γ is

$$t_0 = \exp x_0 = (1 : \omega : \omega^2 : \cdots : \omega^{\ell-1}) \in \mathbb{T}^\ell/\mathbb{T}$$

where $\omega = \exp(2\pi i/\ell)$. The $\overline{L}$-group attached to t_0 is

$$\overline{L}(t_0) =< \omega^{\mathrm{val}_F} >= \mathbb{Z}/\ell\mathbb{Z}$$

and the X-group is trivial, so that the R-group is given by

$$R(t_0) = \mathbb{Z}/\ell\mathbb{Z}.$$

The ℓ irreducible constituents of $\mathrm{Ind}_{\mathcal{B}}^{\mathcal{G}}(t_0)$ are *elliptic* representations [5].

Let $\overline{A}//H_A$ denote the extended quotient of $\overline{A}$ by H_A. The canonical projection

$$\pi : \overline{A}//H_A \to \overline{A}/H_A$$

is a model of reducibility at the point t_0. The fibre $\pi^{-1}(t_0)$ contains ℓ points. This is very much in the spirit of the recent geometric conjecture in [2].

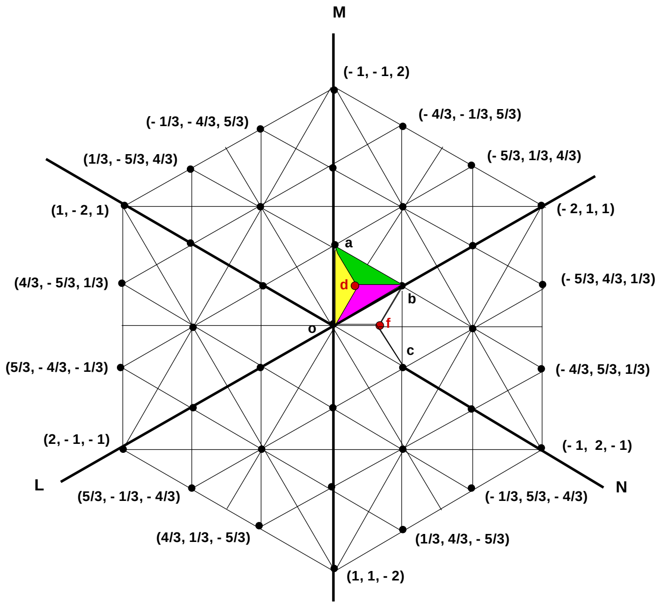
Note that $f = \ell$ as required. The unramified unitary principal series of $\mathrm{SL}(\ell)$ admits a unique L-packet with ℓ elliptic constituents, and this is detected by K_0. For SL(2) we have $K_0 = \mathbb{Z} \oplus \mathbb{Z},\ K_1 = 0$.

Let $\mathcal{G} = \mathrm{SL}(3)$. Then the vector space $\mathfrak{t}$ is the Euclidean plane $\mathbb{R}^2$. The singular hyperplanes [4, Ch. IX, p.325] tessellate $\mathbb{R}^2$ into equilateral triangles, as in Figures 1 and 2. This tessellation of the plane appears in [1, p.104] as the Stiefel diagram for $\mathrm{SU}(3, \mathbb{C})$. The interior of each equilateral triangle is an *alcove*. The closure of an alcove A is denoted $\overline{A}$. The affine Weyl group W_a operates simply transitively on the set of alcoves, and $\overline{A}$ is a fundamental domain for the action of W_a on $\mathfrak{t}$, see [4, p.326]. There is a finer tessellation into isosceles triangles, as in Figures 1 and 2. An isosceles triangle in this tessellation will be denoted Δ. The extended affine Weyl group W_a' acts simply transitively on the set of such isosceles triangles Δ, but the closure $\overline{\Delta}$ is not a fundamental domain for the action of W_a'. Quotient spaces are as follows:

$$\mathfrak{t}/W_a' \simeq \overline{A}/H_A$$

The abelian group H_A is the cyclic group $\mathbb{Z}/3\mathbb{Z}$ which acts on $\overline{A}$ by rotation through $2\pi/3$. If $\overline{A} = oab$ then the special points of $\overline{A}$ are the vertices $\{o, a, b\}$. The quotient $\overline{A}/H_A$ is an identification space of $\overline{\Delta}$: identify two adjacent edges of $\overline{\Delta}$. This space is contractible.

Consider the closed set $oabc$ in Figure 1. This closed set is the closure of the union of two adjacent alcoves A_1, A_2 and is also the closure of the union of 6 isosceles triangles $\Delta_1, \ldots, \Delta_6$. The torus T is an identification space of the closed set $oabc$: identify oa with cb and oc with ab.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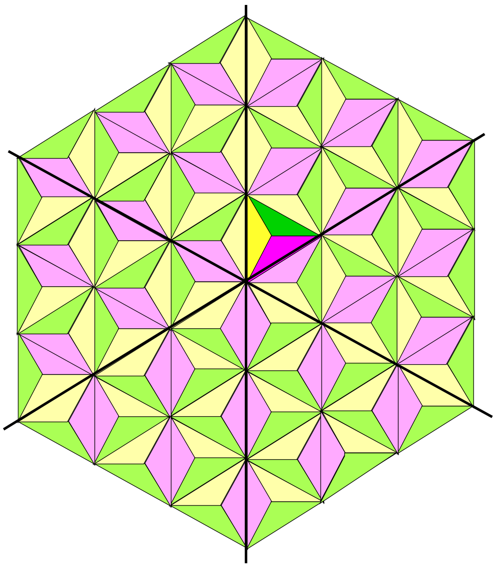

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