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Weijian Zhang and Nicholas J. Higham

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Matrix Depot: An Extensible Test Matrix Collection for Julia*

Weijian Zhang and Nicholas J. Higham†

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Abstract

Matrix Depot is a Julia software package that provides easy access to a large and diverse collection of test matrices. Its novelty is threefold. First, it is extensible by the user, and so can be adapted to include the user's own test problems. In doing so it facilitates experimentation and makes it easier to carry out reproducible research. Second, it amalgamates in a single framework three different types of matrix collections, comprising parametrized test matrices, regularization test problems, and real-life sparse matrix data. Third, it fully exploits the Julia language. It uses multiple dispatch to help provide a simple interface and, in particular, to allow matrices to be generated in any of the numeric data types supported by the language.

Keywords: Julia; software package; test matrices; test problems; matrix algorithm.

1 Introduction

In 1969 Gregory and Karney published a book of test matrices [14]. They stated that "In order to test the accuracy of computer programs for solving numerical problems, one needs numerical examples with known solutions. The aim of this monograph is to provide the reader with suitable examples for testing algorithms for finding the inverses, eigenvalues, and eigenvectors of matrix." At that time it was common for journal papers to be devoted to introducing and analyzing a particular test matrix or class of matrices, examples being the papers of Clement [7] (in the first issue of SIAM Review), Pei [24] (occupying just a quarter of a page), and Gear [13].

Today, test matrices remain of great interest, but not for the same reasons as fifty years ago. Testing accuracy using problems with known solutions is less common because a reference solution correct to machine precision can usually be computed at higher precision without difficulty. The main uses of test matrices nowadays are for exploring the behavior of mathematical quantities (such as eigenvalue bounds) and for measuring the performance of one or more algorithms with respect to accuracy, stability, convergence rate, speed, or robustness.

Various collections of matrices have been made available in software. As well as giving easy access to matrices these collections have the advantage of facilitating reproducibility of experiments [10], whether by the same researcher months later or by different researchers.

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†School of Mathematics, University of Manchester, Manchester, M13 9PL, England (weijian.zhang@manchester.ac.uk, nick.higham@manchester.ac.uk).

An early collection of parametrizable matrices was given by Higham [20] and made available in MATLAB form. The collection was later extended and distributed as a MATLAB toolbox [21]. Many of the matrices in the toolbox were subsequently incorporated into the MATLAB `gallery` function. Marques, Vömel, Demmel, and Parlett [23] present test matrices for tridiagonal eigenvalue problems (already recognized as important by Gregory and Karney, who devoted the last chapter of their book to such matrices). The Harwell–Boeing collection of sparse matrices [11] has been widely used, and is incorporated in the University of Florida Sparse Matrix Collection [8], [9], which contains over 2700 matrices from practical applications, including standard and generalized eigenvalue problems from [1]. Among other MATLAB toolboxes we mention the CONTEST toolbox [25], which produces adjacency matrices describing random networks, and the NLEVP collection of nonlinear eigenvalue problems [2].

The purpose of this work is to provide a test matrix collection for Julia [3], [4], a new dynamic programming language for technical computing. The collection, called Matrix Depot, exploits Julia's multiple dispatch features to enable all matrices to be accessed by one simple interface. Moreover, Matrix Depot is extensible. Users can add matrices from the University of Florida Sparse Matrix Collection and Matrix Market; they can code new matrix generators and incorporate them into Matrix Depot; and they can define new groups of matrices that give easy access to subsets of matrices. The matrices can be generated in any appropriate numeric data type, such as

- floating-point types `Float16` (half precision: 16 bits), `Float32` (single precision: 32 bits), and `Float64` (double precision: 64 bits);
- integer types `Int32` (signed 32-bit integers), `UInt32` (unsigned 32-bit integers), `Int64` (signed 64-bit integers), and `UInt64` (unsigned 32-bit integers);
- `Complex`, where the real and imaginary parts are of any `Real` type (the same for both);
- `Rational` (ratio of integers); and
- arbitrary precision type `BigFloat` (with default precision 256 bits), which uses the GNU MPFR Library [12].

This paper is organized as follows. We start by giving a brief demonstration of Matrix Depot in Section 2. Then we explain the design and implementation of Matrix Depot in Section 3, giving details on how multiple dispatch is exploited, how the collection is stored, accessed, and documented, and how it can be extended. In Section 4 we describe the three classes of matrices in Matrix Depot: parametrized test matrices, regularization test problems, and real-life sparse matrix data. Concluding remarks are given in Section 5.

2 A Taste of Matrix Depot

To download Matrix Depot, in a Julia REPL (read-eval-print loop) run the command

```
> Pkg.add("MatrixDepot")
```

Then import Matrix Depot into the local scope.

```
> using MatrixDepot
```

Now the package is ready to be used. First, we find out what matrices are in Matrix Depot.

```
> matrixdepot()

Matrices:
   1) baart             2) binomial          3) blur              4) cauchy
   5) chebspec          6) chow              7) circul            8) clement
   9) companion        10) deriv2           11) dingdong         12) fiedler
  13) forsythe         14) foxgood          15) frank            16) golub
  17) gravity          18) grcar            19) hadamard         20) hankel
  21) heat             22) hilb             23) invhilb          24) invol
  25) kahan            26) kms              27) lehmer           28) lotkin
  29) magic            30) minij            31) moler            32) neumann
  33) oscillate        34) parter           35) pascal           36) pei
  37) phillips         38) poisson          39) prolate          40) randcorr
  41) rando            42) randsvd          43) rohess           44) rosser
  45) sampling         46) shaw             47) spikes           48) toeplitz
  49) tridiag          50) triw             51) ursell           52) vand
  53) wathen           54) wilkinson        55) wing
Groups:
  all            data           eigen          ill-cond
  inverse        pos-def        random         regprob
  sparse         symmetric
```

All the matrices and groups in the collection are shown. It is also possible to obtain just the list of matrix names.

```
> matrixdepot("all")
55-element Array{ASCIIString,1}:
 "baart"
 "binomial"
 "blur"
 "cauchy"
 "chebspec"
 "chow"
 "circul"
 "clement"
 ...
 "spikes"
 "toeplitz"
 "tridiag"
 "triw"
 "ursell"
 "vand"
 "wathen"
 "wilkinson"
 "wing"
```

Here, "..." denotes that we have omitted some of the output in order to save space. Next, we check the input options of the Hilbert matrix `hilb`.

```
> matrixdepot("hilb")
     Hilbert matrix
    ================

  The Hilbert matrix has (i,j) element 1/(i+j-1). It is notorious for being
  ill conditioned. It is symmetric positive definite and totally positive.
```

```
  Input options:

    *  [type,] dim: the dimension of the matrix.

    *  [type,] row_dim, col_dim: the row and column dimensions.

  Groups: ["inverse", "ill-cond", "symmetric", "pos-def"]

  References:

  M. D. Choi, Tricks or treats with the Hilbert matrix, Amer. Math. Monthly,
  90 (1983), pp. 301-312.

  N. J. Higham, Accuracy and Stability of Numerical Algorithms, second
  edition, Society for Industrial and Applied Mathematics, Philadelphia, PA,
  USA, 2002; sec. 28.1.
```

Note that an optional first argument `type` can be given; it defaults to `Float64`. The string of equals signs on the third line in the output above is Markdown notation for a header. Julia interprets Markdown within documentation, though as we are using typewriter font for code examples here, we display the uninterpreted source. We generate a 4×6 Hilbert matrix with elements in the default double precision type and then in `Rational` type.

```
> matrixdepot("hilb", 4, 6)
4x6 Array{Float64,2}:
 1.0       0.5       0.333333  0.25      0.2       0.166667
 0.5       0.333333  0.25      0.2       0.166667  0.142857
 0.333333  0.25      0.2       0.166667  0.142857  0.125
 0.25      0.2       0.166667  0.142857  0.125     0.111111

> matrixdepot("hilb", Rational, 4, 6)
4x6 Array{Rational{T<:Integer},2}:
 1//1  1//2  1//3  1//4  1//5  1//6
 1//2  1//3  1//4  1//5  1//6  1//7
 1//3  1//4  1//5  1//6  1//7  1//8
 1//4  1//5  1//6  1//7  1//8  1//9
```

A list of all the symmetric matrices in the collection is readily obtained.

```
> matrixdepot("symmetric")
21-element Array{ASCIIString,1}:
 "cauchy"
 "circul"
 "clement"
 "dingdong"
 "fiedler"
 "hankel"
 "hilb"
 "invhilb"
 "kms"
 "lehmer"
 "minij"
 "moler"
 "oscillate"
```

```
 "pascal"
 "pei"
 "poisson"
 "prolate"
 "randcorr"
 "tridiag"
 "wathen"
 "wilkinson"
```

Here, `symmetric` is one of several predefined groups, and multiple groups can be intersected. For example, the `for` loop below prints the smallest and largest eigenvalues of all the 4×4 matrices in Matrix Depot that are symmetric positive definite and (potentially) ill conditioned.

```
> for name in matrixdepot("symmetric", "pos-def", "ill-cond")
      A = full(matrixdepot(name, 4))
      @printf "%9s: smallest eigval = %0.3e, largest eigval = %0.3e\n"
      name eigmin(A) eigmax(A)
  end

   cauchy: smallest eigval = 2.131e-05, largest eigval = 9.776e-01
     hilb: smallest eigval = 9.670e-05, largest eigval = 1.500e+00
  invhilb: smallest eigval = 6.666e-01, largest eigval = 1.034e+04
      kms: smallest eigval = 3.750e-01, largest eigval = 2.086e+00
    moler: smallest eigval = 3.336e-02, largest eigval = 5.122e+00
oscillate: smallest eigval = 1.490e-08, largest eigval = 1.000e+00
   pascal: smallest eigval = 3.802e-02, largest eigval = 2.630e+01
      pei: smallest eigval = 1.000e+00, largest eigval = 5.000e+00
  tridiag: smallest eigval = 3.820e-01, largest eigval = 3.618e+00
```

Matrices can also be accessed by number within the alphabetical list of matrix names.

```
> matrixdepot(2)
"binomial"

> matrixdepot(2:5)
4-element Array{ASCIIString,1}:
 "binomial"
 "cauchy"
 "chebspec"
 "chow"

> matrixdepot(15:20, 5, 6, 1:3)
11-element Array{AbstractString,1}:
 "frank"
 "golub"
 "gravity"
 "grcar"
 "hadamard"
 "hankel"
 "chebspec"
 "chow"
 "baart"
 "binomial"
 "blur"
```

Access by number provides a convenient way to run a test on subsets of matrices in the collection. However, the number assigned to a matrix may change if we include new matrices in the collection. In order to run tests in a way that is repeatable in the future it is best to group matrices into subsets using the macro `@addgroup`. For example, the following command will group test matrices `frank`, `golub`, `gravity`, `grcar`, `hadamard`, `hankel`, `chebspec`, `chow`, `baart`, `binomial`, and `blur` into `test1`.

```
> @addgroup test1 = matrixdepot(15:20, 5, 6, 1:3)
```

After reloading the package, we can run tests on these matrices using group `test1`. Since `blur` (an image deblurring test problem) generates a sparse matrix and the matrix 2-norm is currently not implemented for sparse matrices in Julia, we use `full` to convert the matrix to dense format.

```
> for name in matrixdepot("test1")
      A = full(matrixdepot(name , 4))
      @printf "%9s has 2-norm %0.3e \n" name norm(A)
  end

    baart has 2-norm 3.192e+00
 binomial has 2-norm 4.576e+00
     blur has 2-norm 8.298e-01
 chebspec has 2-norm 6.474e+00
     chow has 2-norm 3.414e+00
    frank has 2-norm 7.624e+00
    golub has 2-norm 2.050e+02
  gravity has 2-norm 6.656e+00
    grcar has 2-norm 2.562e+00
 hadamard has 2-norm 2.000e+00
   hankel has 2-norm 1.160e+01
```

To download the test matrix `SNAP/web-Google` from the University of Florida sparse matrix collection (see Section 4.3 for more details), we first download the data with

```
> matrixdepot("SNAP/web-Google", :get)
```

and then generate the matrix with

```
> matrixdepot("SNAP/web-Google", :r)
916428x916428 sparse matrix with 5105039 Float64 entries:
        [11343 ,      1]  =  1.0
        [11928 ,      1]  =  1.0
        [15902 ,      1]  =  1.0
        [29547 ,      1]  =  1.0
        [30282 ,      1]  =  1.0
        [31301 ,      1]  =  1.0
        [38717 ,      1]  =  1.0
        ...
        [720325, 916427]  =  1.0
        [772226, 916427]  =  1.0
        [785097, 916427]  =  1.0
        [788476, 916427]  =  1.0
        [822938, 916427]  =  1.0
        [833616, 916427]  =  1.0
        [417498, 916428]  =  1.0
```

```
        [843845, 916428]  =  1.0
```

Note that the omission marked "..." was in this case automatically done by Julia based on the height of the terminal window. Matrices loaded in this way are inserted into the list of available matrices, and assigned a number. After downloading further matrices `HB/1138_bus`, `HB/494_bus`, and `Bova/rma10` the list of matrices is as follows.

```
> matrixdepot()

Matrices:
   1) baart            2) binomial         3) blur             4) cauchy
   5) chebspec         6) chow             7) circul           8) clement
   9) companion       10) deriv2          11) dingdong        12) fiedler
  13) forsythe        14) foxgood         15) frank           16) golub
  17) gravity         18) grcar           19) hadamard        20) hankel
  21) heat            22) hilb            23) invhilb         24) invol
  25) kahan           26) kms             27) lehmer          28) lotkin
  29) magic           30) minij           31) moler           32) neumann
  33) oscillate       34) parter          35) pascal          36) pei
  37) phillips        38) poisson         39) prolate         40) randcorr
  41) rando           42) randsvd         43) rohess          44) rosser
  45) sampling        46) shaw            47) spikes          48) toeplitz
  49) tridiag         50) triw            51) ursell          52) vand
  53) wathen          54) wilkinson       55) wing            56) Bova/rma10
  57) HB/1138_bus     58) HB/494_bus      59) SNAP/web-Google

Groups:
  all           data          eigen         ill-cond
  inverse       pos-def       random        regprob
  sparse        symmetric     test1
```

3 Package Design and Implementation

In this section we describe the design and implementation of Matrix Depot, focusing particularly on the novel aspects of exploitation of multiple dispatch, extensibility of the collection, and user-definable grouping of matrices.

3.1 Exploiting Multiple Dispatch

Matrix Depot makes use of multiple dispatch in Julia, an object-oriented paradigm in which the selection of a function implementation is based on the types of each argument of the function. The generic function `matrixdepot` has eight different methods, where each method itself is a function that handles a specific case. This is neater and more convenient than writing eight "case" statements, as is necessary in many other languages.

```
> methods(matrixdepot)
# 8 methods for generic function "matrixdepot":
matrixdepot() ...
matrixdepot(name::AbstractString) ...
matrixdepot(name::AbstractString, method::Symbol) ...
matrixdepot(props::AbstractString...) ...
```

```
matrixdepot(name::AbstractString, args...) ...
matrixdepot(num::Integer) ...
matrixdepot(ur::UnitRange{T<:Real}) ...
matrixdepot(vs::Union{Integer,UnitRange{T<:Real}}...) ...
```

For example, the following two functions are used for accessing matrices by number and range respectively, where `matrix_name_list()` returns a list of matrix names. The second function calls the first function in the inner loop.

```
function matrixdepot(num::Integer)
    matrixstrings = matrix_name_list()
    n = length(matrixstrings)
    if num > n
        error("There are $(n) parameterized matrices,
               but you asked for the $(num)-th.")
    end
    return matrixstrings[num]
end

function matrixdepot(ur::UnitRange)
    matrixnamelist = AbstractString[]
    for i in ur
        push!(matrixnamelist, matrixdepot(i))
    end
    return matrixnamelist
end
```

As a result, `matrixdepot` is a versatile function that can be used for a variety of purposes, including returning matrix information and generating matrices from various input parameters.

In the following example we see how multiple dispatch handles different numbers and types of arguments for the Cauchy matrix.

```
> matrixdepot("cauchy")
     Cauchy matrix
     =============

  Given two vectors x and y, the (i,j) entry of the Cauchy matrix is
  1/(x[i]+y[j]).

  Input options:

    *  [type,] x, y: two vectors.

    *  [type,] x: a vector. y defaults to x.

    *  [type,] dim: the dimension of the matrix. x and y default to [1:dim;].

  Groups: ["inverse", "ill-cond", "symmetric", "pos-def"]

  References:

  N. J. Higham, Accuracy and Stability of Numerical Algorithms, second
  edition, Society for Industrial and Applied Mathematics, Philadelphia, PA,
  USA, 2002; sec. 28.1
```

```
> matrixdepot("cauchy", [1, 2, 3], [4, 5, 6])
3x3 Array{Float64,2}:
 0.2       0.166667  0.142857
 0.166667  0.142857  0.125
 0.142857  0.125     0.111111

> matrixdepot("cauchy", [0.2, 0.3, 0.4])
3x3 Array{Float64,2}:
 2.5      2.0      1.66667
 2.0      1.66667  1.42857
 1.66667  1.42857  1.25

> matrixdepot("cauchy", 3)
3x3 Array{Float64,2}:
 0.5       0.333333  0.25
 0.333333  0.25      0.2
 0.25      0.2       0.166667

> matrixdepot("cauchy", Float32, 3)
3x3 Array{Float32,2}:
 0.5       0.333333  0.25
 0.333333  0.25      0.2
 0.25      0.2       0.166667
```

Multiple dispatch is also exploited in programming the matrices. For example, the Hilbert matrix is implemented as

```
function hilb{T}(::Type{T}, m::Integer, n::Integer)
    H = zeros(T, m, n)
    for j = 1:n, i = 1:m
        @inbounds H[i,j] = one(T)/ (i + j - one(T))
    end
    return H
end
hilb{T}(::Type{T}, n::Integer) = hilb(T, n, n)
hilb(args...) = hilb(Float64, args...)
```

The function `hilb` has three methods, which enable one to request, for example, `hilb(4,2)` for a 4×2 Hilbert matrix of type `Float64`, or simply (thanks to the final two lines) `hilb(4)` for a 4×4 Hilbert matrix of type `Float64`. The keyword `@inbounds` tells Julia to turn off bounds checking in the following expression, in order to speed up execution. Note that in Julia it is not necessary to vectorize code to achieve good performance [3].

All the matrices in Matrix Depot can be generated using the function call

```
matrixdepot("matrix_name", p1, p2, ...),
```

where `matrix_name` is the name of the test matrix, and `p1`, `p2`, ..., are input arguments depending on `matrix_name`. The help comments for each matrix can be viewed by calling function `matrixdepot("matrix_name")`. We can access the list of matrix names by number, range, or a mixture of numbers and range.

1. `matrixdepot(i)` returns the name of the ith matrix;

2. `matrixdepot(i:j)` returns the names of the ith to jth matrices, where $i < j$;

3. `matrixdepot(i:j, k, m)` returns the names of the ith, $(i+1)$st, ..., jth, kth, and mth matrices.

3.2 Matrix Representation

Matrix names in Matrix Depot are represented by Julia strings. For example, the Cauchy matrix is represented by `"cauchy"`. Matrix names and matrix groups are stored as hash tables (`Dict`). In particular, there is a hash table `matrixdict` that maps each matrix name to its underlying function and a hash table `matrixclass` that maps each group to its members.

The majority of parametrized matrices are dense matrices of type `Array{T,2}`, where `T` is the element type of the matrix. Variables of the `Array` type are stored in column-major order. A few matrices are stored as sparse matrices (see also `matrixdepot("sparse")`), in the Compressed Sparse Column (CSC) format; these include `neumann` (a singular matrix from the discrete Neumann problem) and `poisson` (a block tridiagonal matrix from Poisson's equation). Tridiagonal matrices are stored in the built-in type `Tridiagonal` which is defined as follows.

```
immutable Tridiagonal{T} <: AbstractMatrix{T}
    dl::Vector{T}    # sub-diagonal
    d::Vector{T}     # diagonal
    du::Vector{T}    # sup-diagonal
    du2::Vector{T}   # supsup-diagonal for pivoting
end
```

3.3 Matrix Groups

A group is a subset of matrices in Matrix Depot. There are ten predefined groups, which are described in Table 1. Each group is represented by a string. For example, the group of random matrices is represented by `"random"`. Matrices can be accessed by group names, as was illustrated in Section 1.

The macro `@addgroup` is used to add a new group of matrices to Matrix Depot and the macro `@rmgroup` removes an added group. All the predefined matrix groups are stored in the hash table `matrixclass`. The macro `@addgroup` essentially adds a new key-value combination to the hash table `usermatrixclass`. Using a separate hash table prevents the user from contaminating the predefined matrix groups.

Being able to create groups is a useful feature for reproducible research [10]. For example, if we have implemented algorithm `alg01` and we used `circul`, `minij`, and `grcar` as test matrices for `alg01`, we could type

```
> @addgroup alg01_group = ["circul", "minij", "grcar"]
```

This adds a new group to Matrix Depot (we need to reload the package to see the changes).

```
> matrixdepot()

Matrices:
   1) baart            2) binomial         3) blur             4) cauchy
   5) chebspec         6) chow             7) circul           8) clement
```

Table 1: Matrix properties.

Group	Description
`all`	All the matrices in the collection.
`data`	The matrix has been downloaded from the University of Florida Sparse Collection or the Matrix Market Collection.
`eigen`	Part of the eigensystem of the matrix is explicitly known.
`ill-cond`	The matrix is ill-conditioned for some parameter values.
`inverse`	The inverse of the matrix is known explicitly.
`pos-def`	The matrix is positive definite for some parameter values.
`random`	The matrix has random entries.
`regprob`	The output is a test problem for regularization methods.
`sparse`	The matrix is sparse.
`symmetric`	The matrix is symmetric for some parameter values.

```
   9) deriv2          10) dingdong       11) fiedler        12) forsythe
  13) foxgood         14) frank          15) gravity        16) grcar
  17) hadamard        18) hankel         19) heat           20) hilb
  21) invhilb         22) invol          23) kahan          24) kms
  25) lehmer          26) lotkin         27) magic          28) minij
  29) moler           30) neumann        31) oscillate      32) parter
  33) pascal          34) pei            35) phillips       36) poisson
  37) prolate         38) randcorr       39) rando          40) randsvd
  41) rohess          42) rosser         43) sampling       44) shaw
  45) toeplitz        46) tridiag        47) triw           48) vand
  49) wathen          50) wilkinson      51) wing           52) Bova/rma10
  53) HB/1138_bus     54) HB/494_bus     55) SNAP/web-Google

Groups:
  all             data            eigen           ill-cond
  inverse         pos-def         random          regprob
  sparse          symmetric       alg01_group
```

We can then run `alg01` on the test matrices by

```
> for name in matrixdepot(alg01_group)
      A = matrixdepot(name, n)  # n is the dimension of the matrix.
      @printf "Test result for %9s is %0.3e" name alg01(A)
  end
```

3.4 Adding New Matrix Generators

Generators are Julia functions that generate test matrices. When Matrix Depot is first loaded, a directory `myMatrixDepot` is created. It contains two files, `group.jl` and `generator.jl`, where `group.jl` is used for storing all the user defined groups (see Section 3.3) and `generator.jl` is used for storing generator declarations.

Julia packages are simply Git repositories[1]. The directory `myMatrixDepot` is untracked by Git,

[1]Git is a free and open source distributed version control system.

so any local changes to files in `myMatrixDepot` do not make the `MatrixDepot` package "dirty". In particular, all the newly defined groups or matrix generators will not be affected when we upgrade to a new version of Matrix Depot. Matrix Depot automatically loads all Julia files in `myMatrixDepot`. This feature allows a user to simply drop generator files into `myMatrixDepot` without worrying about how to link them to Matrix Depot.

A new generator is declared using the syntax `include_generator(FunctionName, "fname", f)`. This adds the new mapping `"fname"` $\rightarrow$ `f` to the hash table `matrixdict`, which we recall maps each matrix name to its underlying function. Matrix Depot will refer to function `f` using string `"fname"` so that we can call function `f` by `matrixdepot("fname"...)`.

For example, suppose we have the following Julia file `rand.jl`, which contains two generators `randsym` and `randorth` and we want to use them from Matrix Depot. The triple quotes in the file delimit the documentation for the functions.

```
"""
random symmetric matrix
=======================

Input options:

* n: the dimension of the matrix
"""
function randsym(n)
  A = zeros(n, n)
  for j = 1:n
      for i = 1:j
          A[i,j] = randn()
          if i != j; A[j,i] = A[i,j] end
      end
  end
  return A
end

"""
random orthogonal matrix
========================

Input options:

* n: the dimension of the matrix
"""
randorth(n) = qr(randn(n,n))[1]
```

We can copy the file `rand.jl` to the directory `myMatrixDepot` and write the following two lines to `generator.jl`.

```
include_generator(FunctionName, "randsym", randsym)
include_generator(FunctionName, "randorth", randorth)
```

This includes the functions `randsym` and `randorth` in Matrix Depot, as we can see by looking at the matrix list.

```
> matrixdepot()
```

```
Matrices:
   1) baart             2) binomial          3) blur              4) cauchy
   5) chebspec          6) chow              7) circul            8) clement
   9) companion        10) deriv2           11) dingdong         12) fiedler
  13) forsythe         14) foxgood          15) frank            16) golub
  17) gravity          18) grcar            19) hadamard         20) hankel
  21) heat             22) hilb             23) invhilb          24) invol
  25) kahan            26) kms              27) lehmer           28) lotkin
  29) magic            30) minij            31) moler            32) neumann
  33) oscillate        34) parter           35) pascal           36) pei
  37) phillips         38) poisson          39) prolate          40) randcorr
  41) rando            42) randorth         43) randsvd          44) randsym
  45) rohess           46) rosser           47) sampling         48) shaw
  49) spikes           50) toeplitz         51) tridiag          52) triw
  53) ursell           54) vand             55) wathen           56) wilkinson
  57) wing
Groups:
  all            data           eigen          ill-cond
  inverse        pos-def        random         regprob
  sparse         symmetric
```

The new generators can be used just like the built-in ones.

```
> matrixdepot("randsym")
     random symmetric matrix
     =======================

    Input options:

     *  n: the dimension of the matrix

> matrixdepot("randsym", 4)
4x4 Array{Float64,2}:
 -0.00992523  0.174531   -1.73322   -0.765096
  0.174531    1.69308     0.269062   0.594058
 -1.73322     0.269062   -0.824277  -0.541458
 -0.765096    0.594058   -0.541458  -0.480428

> matrixdepot("randorth")
     random orthogonal matrix
     ========================

    Input options:

     *  n: the dimension of the matrix

> A = matrixdepot("randorth", 4)
4x4 Array{Float64,2}:
 -0.233943   0.179893   0.563926  -0.771295
 -0.769649  -0.141938  -0.5807    -0.224235
  0.247165   0.832118  -0.449941  -0.20986
 -0.540204   0.505046   0.377263   0.557477

> A'*A - eye(4,4)
4x4 Array{Float64,2}:
 -2.22045e-16   1.66533e-16  -2.77556e-17  -1.66533e-16
```

```
  1.66533e-16   -1.11022e-16   -3.05311e-16    1.66533e-16
 -2.77556e-17   -3.05311e-16   -1.11022e-16    1.94289e-16
 -1.66533e-16    1.66533e-16    1.94289e-16    0.0
```

We can also add group information with the function `include_generator`. The following lines are put in `generator.jl`.

```
include_generator(Group, "random", randsym)
include_generator(Group, "random", randorth)
```

This adds the functions `randsym` and `randorth` to the group `random`, as we can see with the following query (after reloading the package).

```
> matrixdepot("random")
10-element Array{ASCIIString,1}:
 "golub"
 "oscillate"
 "randcorr"
 "rando"
 "randorth"
 "randsvd"
 "randsym"
 "rohess"
 "rosser"
 "wathen"
```

3.5 Documentation

The Matrix Depot documentation is created using the documentation generator Sphinx[2] and is hosted at Read the Docs[3]. Its primary goals are to provide examples of usage of Matrix Depot and to give a brief summary of each matrix in the collection. Matrices are listed alphabetically with hyperlinks to the documentation for each matrix. Most parametrized matrices are presented with heat map plots, which are produced using the Winston package[4], with the color range determined by the smallest and largest entries of the matrix. For example, Figure 1 shows how the Wathen matrix is documented in Matrix Depot.

4 The Matrices

We now describe the matrices that are provided with, or can be downloaded into, Matrix Depot.

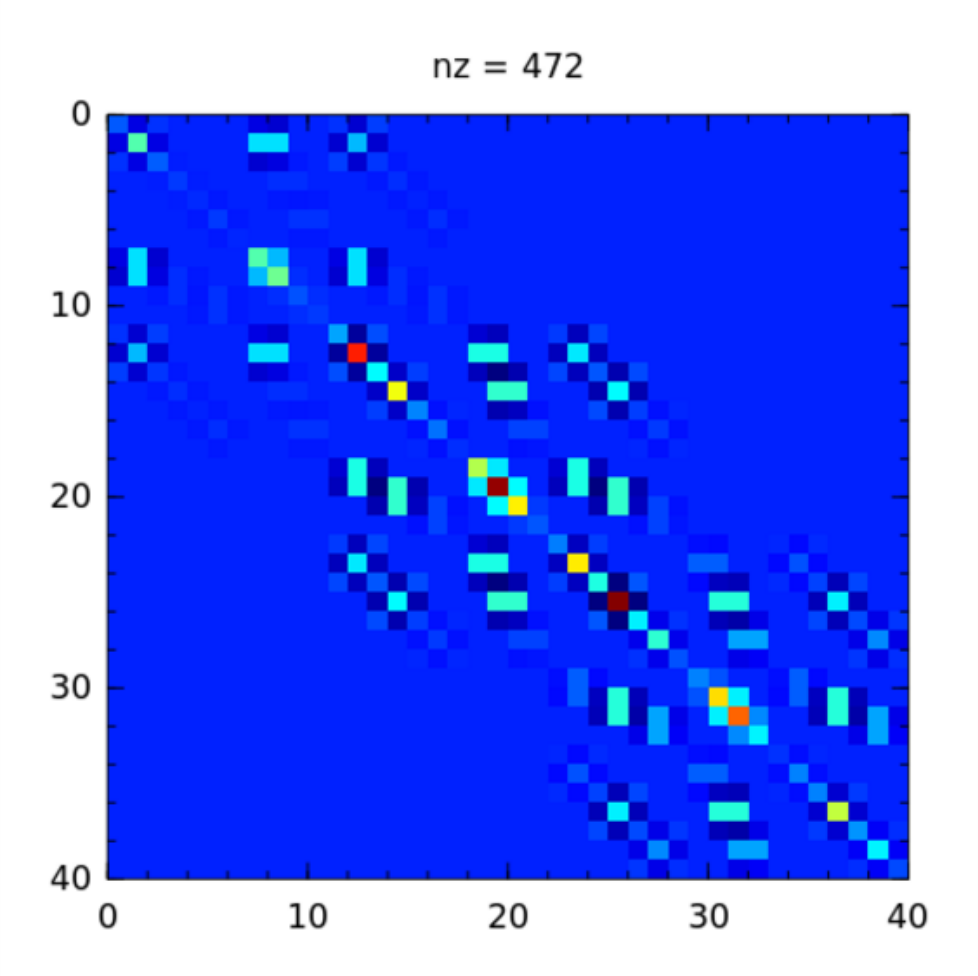
4.1 Parametrized Matrices

In Matrix Depot v0.5.2, there are 55 parametrized matrices (including the regularization problems described in the next section), most of which originate from the Test Matrix Toolbox [21]. All these matrices can be generated as `matrixdepot("matrix_name", n)`, where `n` is the dimension of the matrix.

[2] http://sphinx-doc.org/
[3] http://matrixdepotjl.readthedocs.org
[4] https://github.com/nolta/Winston.jl

wathen

The Wathen matrix is a sparse, symmetric positive, random matrix arising from the finite element method [wath87]. It is the consistent mass matrix for a regular *nx-by-ny* grid of 8-node elements.



[wath87] A.J. Wathen, Realistic eigenvalue bounds for the Galerkin mass matrix, IMA J. Numer. Anal., 7 (1987), pp. 449-457.

Figure 1: Documentation for the Wathen matrix

Many matrices can have more than one input parameter, and multiple dispatch provides a convenient mechanism for taking different actions for different argument types. For example, the `tridiag` function generates a tridiagonal matrix from vector arguments giving the subdiagonal, diagonal, and superdiagonal vectors, but a tridiagonal Toeplitz matrix can be obtained by supplying scalar arguments that specify the dimension of the matrix, the subdiagonal, the diagonal, and the superdiagonal. If a single, scalar argument `n` is supplied then an `n`-by-`n` tridiagonal Toeplitz matrix with subdiagonal and superdiagonal -1 and diagonal 2 is constructed. This matrix arises in applying central differences to a second derivative operator, and the inverse and the condition number are known explicitly [22, sec. 28.5].

Here is an example of the different usages of `tridiag`.

```
> matrixdepot("tridiag")
     Tridiagonal Matrix
    ====================

  Construct a tridiagonal matrix of type Tridiagonal.

  Input options:

    *  [type,] v1, v2, v3: v1 and v3 are vectors of subdiagonal and
      superdiagonal elements, respectively, and v2 is a vector of diagonal
      elements.

    *  [type,] dim, x, y, z: dim is the dimension of the matrix, x, y, z are
      scalars. x and z are the subdiagonal and superdiagonal elements,
      respectively, and y is the diagonal elements.

    *  [type,] dim: x = -1, y = 2, z = -1. This matrix is also known as the
      second difference matrix.

  Groups: ["inverse", "ill-cond", "pos-def", "eigen"]

  References:

  J. Todd, Basic Numerical Mathematics, Vol. 2: Numerical Algebra, Birkhauser,
  Basel, and Academic Press, New York, 1977, p. 155.


> matrixdepot("tridiag", [2,5,6;], ones(4), [3,4,1;])
4x4 Tridiagonal{Float64}:
 1.0  3.0  0.0  0.0
 2.0  1.0  4.0  0.0
 0.0  5.0  1.0  1.0
 0.0  0.0  6.0  1.0

> matrixdepot("tridiag", 4, 5, 3, 1)
4x4 Tridiagonal{Float64}:
 3.0  1.0  0.0  0.0
 5.0  3.0  1.0  0.0
 0.0  5.0  3.0  1.0
 0.0  0.0  5.0  3.0

> matrixdepot("tridiag", Int, 4)
4x4 Tridiagonal{Int64}:
  2  -1   0   0
```

```
 -1    2   -1    0
  0   -1    2   -1
  0    0   -1    2
```

4.2 Test Problems for Regularization Methods

A mathematical problem is ill-posed if the solution is not unique or if an arbitrarily small perturbation of the data can cause an arbitrarily large change in the solution. Regularization methods are an important class of methods for dealing with such problems [16], [19]. One means of generating test problems for regularization methods is to discretize a given ill-posed problem.

Matrix Depot contains a group of regularization test problems derived from Hansen's MATLAB Regularization Tools [15], [17], [18] that are mostly discretizations of Fredholm integral equations of the first kind:

$$\int_0^1 K(s,t)f(t)\,dt = g(s), \qquad 0 \le s \le 1.$$

The regularization test problems form the group `regprob`.

```
> matrixdepot("regprob")
11-element Array{ASCIIString,1}:
 "baart"
 "blur"
 "deriv2"
 "foxgood"
 "gravity"
 "heat"
 "phillips"
 "shaw"
 "spikes"
 "ursell"
 "wing"
```

Each problem is a linear system $Ax = b$ where the matrix A and vectors x and b are obtained by discretization (using quadrature or the Galerkin method) of K, f, and g. By default, we generate only A, which is an ill-conditioned matrix. The whole test problem will be generated if the parameter `matrixonly` is set to `false`, and in this case the output has type `RegProb`, which is defined as

```
immutable RegProb{T}
  A::AbstractMatrix{T}  # matrix of interest
  b::AbstractVector{T}  # right-hand side
  x::AbstractVector{T}  # the solution to Ax = b
end
```

If `r` is a generated test problem, then `r.A`, `r.b`, and `r.x` are the matrix A and vector x and b respectively. For example, the test problem `wing` can be generated as follows.

```
> matrixdepot("wing")
     A Problem with a Discontinuous Solution
    ========================================

  Input options:
```

```
    *  [type,] n, t1, t2, [matrixonly]: the dimension of matrix is n. t1 and
       t2 are two real scalars such that 0 < t1 < t2 < 1. If matrixonly =
       false, the linear system A, b, x will be generated (matrixonly = true
       by default).

    *  [type,] n, [matrixonly]: t1 = 1/3 and t2 = 2/3.

  Groups: ["regprob"]

  References:

  G. M. Wing, A Primer on Integral Equations of the First Kind, Society for
  Industrial and Applied Mathematics, 1991, p. 109.

> A = matrixdepot("wing", 4)
4x4 Array{Float64,2}:
 0.031189   0.0921165  0.148804  0.198786
 0.0310674  0.0889342  0.134959  0.164156
 0.0309463  0.085862   0.122403  0.13556
 0.0308257  0.0828958  0.111014  0.111945

> r = matrixdepot("wing", 4, false)
Test problems for Regularization Methods
A:
4x4 Array{Float64,2}:
 0.031189   0.0921165  0.148804  0.198786
 0.0310674  0.0889342  0.134959  0.164156
 0.0309463  0.085862   0.122403  0.13556
 0.0308257  0.0828958  0.111014  0.111945
b:
4-element Array{Float64,1}:
 0.0804953
 0.0751385
 0.0701787
 0.0655842
x:
4-element Array{Float64,1}:
 0.0
 0.5
 0.5
 0.0

> r.x
4-element Array{Float64,1}:
 0.0
 0.5
 0.5
 0.0
```

4.3 Matrix Data from External Sources

Matrix Depot provides access to matrices from Matrix Market [5] and the University of Florida Sparse Matrix Collection [8], [9], both of which contain many matrices taken from applications.

In particular, these sources contain many large, sparse matrices.

Matrix Market and the University of Florida Sparse Matrix Collection both categorize matrices by application domain and the problem source and both provide matrices in Matrix Market Format [6]. These similarities allow us to design a generic interface for both collections. The symbol `:get` (or `:g`) is used for downloading matrices from both collections and the symbol `:read` (or `:r`) is used for reading in matrices already downloaded. Downloaded matrix data is stored on disk in the Matrix Market format and when read into Julia is stored in the type `SparseMatrixCSC`.

`MatrixDepot.update()` downloads the matrix name data files from the two web servers.

```
> MatrixDepot.update()
  % Total    % Received % Xferd  Average Speed   Time    Time     Time  Current
                                 Dload  Upload   Total   Spent    Left  Speed
100 1887k    0 1887k    0     0  97337       0 --:--:--  0:00:19 --:--:--  472k
  % Total    % Received % Xferd  Average Speed   Time    Time     Time  Current
                                 Dload  Upload   Total   Spent    Left  Speed
100 41552    0 41552    0     0   4421       0 --:--:--  0:00:09 --:--:-- 41018
```

The UF Sparse Matrix Collection is divided into matrix groups and the group of a matrix forms part of the full name of the matrix [9]. For example, the full name of the matrix `1138_bus` in the Harwell-Boeing Collection is `HB/1138_bus`.

```
> matrixdepot("HB/1138_bus", :get)
  % Total    % Received % Xferd  Average Speed   Time    Time     Time  Current
                                 Dload  Upload   Total   Spent    Left  Speed
100 19829  100 19829    0     0   2320       0  0:00:08  0:00:08 --:--:-- 49572

> matrixdepot("HB/1138_bus", :read)
1138x1138 Symmetric{Float64,SparseMatrixCSC{Float64,Int64}}:
 1474.78       0.0        0.0    ...    0.0        0.0          0.0     0.0
    0.0        9.13665    0.0           0.0        0.0          0.0     0.0
    0.0        0.0       69.6147        0.0        0.0          0.0     0.0
    0.0        0.0        0.0           0.0        0.0          0.0     0.0
   -9.01713    0.0        0.0           0.0        0.0          0.0     0.0
    0.0        0.0        0.0    ...    0.0        0.0          0.0     0.0
    0.0        0.0        0.0           0.0        0.0          0.0     0.0
    0.0        0.0        0.0           0.0        0.0          0.0     0.0
    0.0        0.0        0.0           0.0        0.0          0.0     0.0
    0.0       -3.40599    0.0           0.0        0.0          0.0     0.0
    ...                                                               ...
    0.0        0.0        0.0           0.0        0.0          0.0     0.0
    0.0        0.0        0.0    ...    0.0      -24.3902       0.0     0.0
    0.0        0.0        0.0           0.0        0.0          0.0     0.0
    0.0        0.0        0.0           0.0        0.0          0.0     0.0
    0.0        0.0        0.0           0.0        0.0          0.0     0.0
    0.0        0.0        0.0          26.5639     0.0          0.0     0.0
    0.0        0.0        0.0    ...    0.0       46.1767       0.0     0.0
    0.0        0.0        0.0           0.0        0.0      10000.0     0.0
    0.0        0.0        0.0           0.0        0.0          0.0   117.647
```

Matrices from the University of Florida Sparse Matrix Collection are stored in `MatrixDepot/data/uf` and they are stored by group (to avoid duplicate names), i.e., one directory per group. Similarly, matrices from Matrix Market are stored in `MatrixDepot/data/mm`. Both directories are untracked by Git.

We can download a whole group of matrices from the University of Florida sparse matrix collection using the command `matrixdepot("group name/*", :get)`. The next example downloads all 67 matrices in the Gset group of matrices from random graphs (contributed by Y. Ye) then displays all the matrices in Matrix Depot, including the newly downloaded matrices.

```
> matrixdepot("Gset/*", :get)

Downloading all matrices in group Gset...
  % Total    % Received % Xferd  Average Speed   Time    Time     Time  Current
                                 Dload  Upload   Total   Spent    Left  Speed
100 48083  100 48083    0     0  95388      0 --:--:-- --:--:-- --:--:-- 96166
download:/home/weijian/.julia/v0.4/MatrixDepot/src/../data/uf/Gset/G1.tar.gz
G1/G1.mtx
  % Total    % Received % Xferd  Average Speed   Time    Time     Time  Current
                                 Dload  Upload   Total   Spent    Left  Speed
100 55180  100 55180    0     0  75318      0 --:--:-- --:--:-- --:--:-- 75692
download:/home/weijian/.julia/v0.4/MatrixDepot/src/../data/uf/Gset/G10.tar.gz
G10/G10.mtx
  % Total    % Received % Xferd  Average Speed   Time    Time     Time  Current
                                 Dload  Upload   Total   Spent    Left  Speed
100  5926  100  5926    0     0  23126      0 --:--:-- --:--:-- --:--:-- 23515
download:/home/weijian/.julia/v0.4/MatrixDepot/src/../data/uf/Gset/G11.tar.gz
G11/G11.mtx
  % Total    % Received % Xferd  Average Speed   Time    Time     Time  Current
                                 Dload  Upload   Total   Spent    Left  Speed
100  6349  100  6349    0     0  24223      0 --:--:-- --:--:-- --:--:-- 24608
...

> matrixdepot()

Matrices:
   1) baart            2) binomial         3) blur             4) cauchy
   5) chebspec         6) chow             7) circul           8) clement
   9) companion       10) deriv2          11) dingdong        12) fiedler
  13) forsythe        14) foxgood         15) frank           16) golub
  17) gravity         18) grcar           19) hadamard        20) hankel
  21) heat            22) hilb            23) invhilb         24) invol
  25) kahan           26) kms             27) lehmer          28) lotkin
  29) magic           30) minij           31) moler           32) neumann
  33) oscillate       34) parter          35) pascal          36) pei
  37) phillips        38) poisson         39) prolate         40) randcorr
  41) rando           42) randsvd         43) rohess          44) rosser
  45) sampling        46) shaw            47) spikes          48) toeplitz
  49) tridiag         50) triw            51) ursell          52) vand
  53) wathen          54) wilkinson       55) wing            56) Gset/G10
  57) Gset/G11        58) Gset/G12        59) Gset/G13        60) Gset/G14
  61) Gset/G15        62) Gset/G16        63) Gset/G17        64) Gset/G18
  65) Gset/G19        66) Gset/G1         67) Gset/G20        68) Gset/G21
  69) Gset/G22        70) Gset/G23        71) Gset/G24        72) Gset/G25
  73) Gset/G26        74) Gset/G27        75) Gset/G28        76) Gset/G29
  77) Gset/G2         78) Gset/G30        79) Gset/G31        80) Gset/G32
  81) Gset/G33        82) Gset/G34        83) Gset/G35        84) Gset/G36
  85) Gset/G37        86) Gset/G38        87) Gset/G39        88) Gset/G3
  89) Gset/G40        90) Gset/G41        91) Gset/G42        92) Gset/G43
  93) Gset/G44        94) Gset/G45        95) Gset/G46        96) Gset/G47
  97) Gset/G48        98) Gset/G49        99) Gset/G4        100) Gset/G50
```

```
 101) Gset/G51        102) Gset/G52        103) Gset/G53        104) Gset/G54
 105) Gset/G55        106) Gset/G56        107) Gset/G57        108) Gset/G58
 109) Gset/G59        110) Gset/G5         111) Gset/G60        112) Gset/G61
 113) Gset/G62        114) Gset/G63        115) Gset/G64        116) Gset/G65
 117) Gset/G66        118) Gset/G67        119) Gset/G6         120) Gset/G7
 121) Gset/G8         122) Gset/G9
Groups:
  all            data           eigen          ill-cond
  inverse        pos-def        random         regprob
  sparse         symmetric      test1
```

The full name of a matrix in Matrix Market comprises three parts: the collection name, the set name, and the matrix name. For example, the full name of the matrix `BCSSTK14` in the set `BCSSTRUC2` from the Harwell-Boeing Collection is `Harwell-Boeing/bcsstruc2/bcsstk14`. Note that both set name and matrix name are in lower case.

```
> matrixdepot("Harwell-Boeing/bcsstruc2/bcsstk14", :get)
  % Total    % Received % Xferd  Average Speed   Time    Time     Time  Current
                                 Dload  Upload   Total   Spent    Left  Speed
100  292k  100  292k    0     0  22635      0  0:00:13  0:00:13 --:--:-- 61144
download:/home/weijian/.julia/v0.4/MatrixDepot/data/mm/Harwell-Boeing/bcsstruc2
/bcsstk14.mtx.gz

> matrixdepot("Harwell-Boeing/bcsstruc2/bcsstk14", :read)
1806x1806 Symmetric{Float64,SparseMatrixCSC{Float64,Int64}}:
      1.93161e6  0.0      -1.02166e5  ...       0.0             0.0
      0.0        1.0       0.0                  0.0             0.0
     -1.02166e5  0.0       1.93147e6           0.0             0.0
 -35568.9        0.0       1.65787e5           0.0             0.0
     -1.06959e5  0.0      -1.06959e5           0.0             0.0
     -1.65835e5  0.0   35568.9         ...      0.0             0.0
   -717.845      0.0       0.0                  0.0             0.0
      0.0        0.0   88998.5                  0.0             0.0
      0.0        0.0      -1.82865e6           0.0             0.0
      0.0        0.0       1.24988e5           0.0             0.0
      ...                                                       ...
      0.0        0.0       0.0                  0.0             0.0
      0.0        0.0       0.0                  1.06103e7      -5.25151e5
      0.0        0.0       0.0                 -5.25151e5  -53434.0
      0.0        0.0       0.0       ...        1.06959e5      -1.65835e5
      0.0        0.0       0.0                  0.0             0.0
      0.0        0.0       0.0                  1.06959e5   35568.9
      0.0        0.0       0.0            -816518.0             1.21311e7
      0.0        0.0       0.0                  4.55624e7       8.15266e5
      0.0        0.0       0.0       ...        8.15266e5       5.27942e8
```

We recommend downloading matrices from the University of Florida sparse matrix collection when there is a choice, because almost every matrix from Matrix Market is included in it.

5 Concluding Remarks

Matrix Depot follows in the footsteps of earlier collections of matrices. Its novelty is threefold. First, it is extensible by the user, and so can be adapted to the user's needs. In doing so it facilitates experimentation, and in particular makes it easier to do reproducible research. Second, it

combines three distinct test matrix collections, namely, parametrized test matrices, regularization test problems, and real-life sparse matrix data, in a single framework. Third, it fully exploits the Julia language. It uses multiple dispatch to help provide a simple interface and, in particular, to allow matrices to be generated in any of the numeric data types supported by the language. Matrix Depot therefore anticipates the development of intrinsic support in Julia for computations with `BigFloat` and other data types.

Matrix Depot has been in development since 2014. It is an open source project[5] hosted on GitHub and is available under the MIT License. A first release was announced in December 2014. Matrix Depot `v0.5.2` is the latest official release and consists of around 3,000 lines of source code, with test coverage of 98.91% according to Codecov[6]. From GitHub traffic analytics, we learn that Matrix Depot has 40 to 70 unique downloads (unique cloners) every month. Matrix Depot also benefits the development of other Julia packages. LightGraphs[7], an optimized graph package for Julia, for example, has embedded Matrix Depot as its database.

We built Matrix Depot to facilitate the development and testing of matrix (and other) algorithms in Julia. and we will continue to develop Matrix Depot by introducing new test matrices and integrating with other test collections.

Acknowledgements

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