Essential partial differential equations

David F. Griffiths, John W. Dold and David J. Silvester

2015

MIMS EPrint: 2016.6

Manchester Institute for Mathematical Sciences
School of Mathematics
The University of Manchester

Reports available from: http://www.manchester.ac.uk/mims/eprints
And by contacting: The MIMS Secretary
School of Mathematics
The University of Manchester
Manchester, M13 9PL, UK

ISSN 1749-9097
## Contents

1 Setting the Scene ........................................ 1
  1.1 Some Classical PDEs .............................. 2
  1.2 … and Some Classical Solutions. ................. 4

2 Boundary and Initial Data ............................... 11
  2.1 Operator Notation ................................. 14
  2.2 Classification of Boundary Value Problems ...... 16
    2.2.1 Linear Problems .......................... 16
    2.2.2 Nonlinear Problems .......................... 20
    2.2.3 Well-Posed Problems ...................... 21

3 The Origin of PDEs ..................................... 27
  3.1 Newton’s Laws .................................. 27
    3.1.1 The Wave Equation for a String .......... 27
  3.2 Conservation Laws .............................. 29
    3.2.1 The Heat Equation ......................... 29
    3.2.2 Laplace’s Equation and the Poisson Equation 31
    3.2.3 The Wave Equation in Water .............. 32
    3.2.4 Burgers’ Equation ......................... 34

4 Classification of PDEs ................................. 37
  4.1 Characteristics of First-Order PDEs ............. 38
  4.2 Characteristics of Second-Order PDEs .......... 44
    4.2.1 Hyperbolic Equations ..................... 45
    4.2.2 Parabolic Equations ....................... 46
    4.2.3 Elliptic Equations ......................... 49
  4.3 Characteristics of Higher-Order PDEs ............ 52
  4.4 Postscript ....................................... 53

5 Boundary Value Problems in \( \mathbb{R}^1 \) ........ 59
  5.1 Qualitative Behaviour of Solutions ............... 61
  5.2 Comparison Principles and Well-Posedness ....... 64
Essential Partial Differential Equations
Analytical and Computational Aspects
Griffiths, D.F.; Dold, J.W.; Silvester, D.J.
2015, XI, 368 p. 106 illus., 1 illus. in color., Softcover
ISBN: 978-3-319-22568-5