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Higham, Nicholas J.

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And by contacting: The MIMS Secretary
School of Mathematics
The University of Manchester
Manchester, M13 9PL, UK

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How to Read and Understand a Paper†

Nicholas J. Higham

Whether you are a mathematician or work in another discipline and need to use mathematical results you will need to read mathematics papers—perhaps lots of them. The purpose of this article is to give advice on how to go about reading mathematics papers and gaining understanding from them.

The advice is particularly aimed at inexperienced readers. A professional mathematician may read from tens to hundreds of papers every year, including published papers, manuscripts sent for refereeing by journals, and draft papers written by students and colleagues. To a large extent the suggestions I make here are ones that you naturally adopt after reading sufficiently many papers.

Mathematics papers fall into two main types: primary research papers and review papers. Review papers give an overview of an area and usually contain a substantial amount of background material. By design they tend to be easier to read than papers presenting new research, although they are often longer. The suggestions in this article apply to both types of paper.

1 The Anatomy of a Paper

Mathematics papers are fairly rigid in format, having some or all of the following components.

Title The title should indicate what the paper is about and give a hint about the paper's contributions.

Abstract The abstract describes the problem being tackled and summarizes the contributions of the paper. The length and the amount of detail both vary greatly. The abstract is meant to be able to stand alone. Often it is visible to everyone on a journal's web site while the paper is only visible to subscribers.

Introduction The first section of the paper, almost always called "Introduction", sets out the context and problem being addressed in more detail than the abstract. Depending on how the paper has been written the introduction may or may not describe the results and conclusions. Some papers lend themselves to a question being posed in the introduction but only fully answered in a conclusions section.

Conclusions Many, but not all, papers contain a final section with a title such as "Conclusion" or "Concluding Remarks" that summarizes the main conclusions of the paper. Omission of such a section indicates that the conclusions have been stated in the introduction or perhaps at the end of a section describing experiments, or that no explicit summary has been provided. This section is often used to identify open questions and describe areas for future research, and such suggestions can be very useful if you are looking for problems to work on.

Appendix Some papers contain one or more appendices, which contain material deemed best separated from the main paper, perhaps because it would otherwise clutter up the development or because it contains tedious details.

References The references section contains a list of publications that are referred to in the text and that the reader might want to consult.

Supplementary Materials A relatively new concept in mathematics is the notion of additional materials that are available on the publisher's website along with the paper but are not actually part of the paper. These might include figures, computer programs, data, and other further material, and might not have been refereed even if the paper itself has. It is not always easy to tell if a paper has supplementary materials, as different journals have different conventions for referring to them. They might be mentioned at the end of the paper or in a footnote on the first page, and may be referred to with "see the supplementary materials" or via an item in the the reference list.

2 Deciding Whether to Read a Paper

A common scenario is that you come across a paper that, based on the title, you think you might need to read. For example, you may be signed up to receive alerts from a journal or search engine and become aware of a new paper on a topic related to your interests. How do you decide whether to read the paper? The abstract should contain enough information about the context of the work and the paper's results for you to make a decision. However, abstracts are sometimes very short and are not always well written, so it

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may be necessary to skim through the introduction and conclusions sections of the paper.

The reference list is worth perusing. If few of the references are familiar this may mean that the paper presents a rather different view on the topic than you expected, perhaps because the authors are from a different field. If papers that you know are relevant are missing this is a warning that the authors may not be fully aware of past work on the problem.

If the main results of the paper are theorems, read those to see whether it is worth spending further time on the paper. Consider also the reputation of the journal and the authors and, unless the paper is very recent, check how often (and how, and by whom) it has been cited in order to get a feel for what other people think about it. (Citations can be checked using online tools, such as Google Scholar or one of several other services. most of which require a subscription.)

3 Getting an Overview

A paper does not have to be read linearly. You may want to make multiple passes, beginning by reading the abstract, introduction, and conclusions, as well as looking at the tables, figures, and references.

Many authors end the introduction with a paragraph that gives an overview of what appears in each part of the paper. Sometimes, though, a glance at the paper's section headings provides a more easily assimilated summary of the content and organization.

Another way in which you might get an overview of the paper is by reading the main results first: the lemmas, theorems, algorithms, and associated definitions, omitting proofs. The usefulness of this approach will depend on the topic and your familiarity with it.

4 Understanding

It is often hard to understand what you are reading. After all, research papers are meant to contain original ideas, and ideas that you have not seen before can be hard to grasp. You may want to stop and ponder an argument, perhaps playing with examples.

I strongly recommend making notes, to help you understand the text and avoid having to retrace your steps in grasping a tricky point if you come back to the paper in the future. It is also a good idea to write a summary of your overall thoughts on the paper; when you go back to the paper a few months or years later your summary will be the first thing to look at. I recommend dating your notes and summary, as in the future it can be useful to know when they were written. Indeed I have papers that I have read several times and the notes show how my understanding changed on each reading. (There exist papers for which multiple readings are needed to appreciate fully the contents: perhaps because the paper is deep, because it is badly written, or both!)

As well as writing notes it is a good idea to mark key sentences, theorems, and so on. I do this either by putting a vertical line in the margin that delineates the area of interest or by marking the relevant text with a highlighter pen.

I write my notes on a hard copy of the paper. Many programs are available that will allow you to annotate PDF files on-screen, though using mathematical notation may be problematic; one solution is to handwrite notes then scan them in and append them to the PDF file.

A good exercise, especially if you are inexperienced at writing papers, is to write your own abstract for the paper (100–200 words, say).

Writing while you read turns you from a passive reader into an active one, and being an active reader helps you to understand and remember the contents. One useful technique is to try out special cases of results. If a theorem is stated for analytic functions, see what it says for polynomials or for the exponential. If a theorem is stated for $n \times n$ matrices check it for $n = 1, 2, 3$. Another approach is to ask yourself what would happen if one of the conditions in a theorem were to be removed: where would the proof break down?

When you reach a point that you do not understand it may be best to jump to the end of the argument and go back over the details later, to avoid getting bogged down. Keep in mind that some ideas and techniques are so well known to researchers in the relevant field that they might not be spelled out. If you are new to the field you may at first need a bit of help from a more experienced colleague to fill in what appear to be gaps in arguments.

It is important to keep in mind that what you are reading may be badly explained or just wrong. Typographical errors are quite common, especially in preprints and in papers that have not been copy edited. Mathematical errors also occur, and even the best journals occasionally have to print corrections ("errata") to previously published articles.

In mathematical writing certain standard phrases are used that have particular meanings. "It follows that" or

"it is easy to see that" mean that the next statement can be proved without using any new ideas and that giving the details would clutter the text. The detail may, however, be tedious. The shorter "hence", "therefore", or "so" imply a more straightforward conclusion. "It can be shown that" again implies that details are not felt to be worth including but is noncommittal about the difficulty of the proof.

5 Documenting Your Reading

I advise keeping a record of which papers you have read, even if you have only read them partially. If you are a beginning PhD student this may seem unnecessary, as at first you will be able to keep the papers in your mind. But at some point you will forget which papers you have read and having this information readily available will be very useful.

A few decades ago papers existed only as hard copies and one would file them by author or subject. Today, most papers are obtained as PDF downloads that can be stored on our computers. Various computer programs are available for managing collections of papers. One of those, or a BibTeX database, can serve to record what you have read and provide links to the PDF files.

6 Screen or Print?

Should you read papers on a computer screen or in print form? This is a personal choice. People brought up in the digital publishing era may be happy reading on-screen, but others, such as me, may feel that they can only properly read a paper in hard copy form. There is no doubt that hard copy allows easier viewing of multiple pages at the same time, while a PDF file makes it easier to search for a particular term and can be zoomed to whatever size is most comfortable to read. It is important to try both and use whatever combination of screen and print works best for you.

If you do read on-screen, keep in mind that most PDF readers allow you to customize the colors. White or yellow text on a black background may be less strain on the eyes than the default black on white. In Adobe Acrobat the colors can be changed with the menu option Preferences-Accessibility-Document Colors Options.

7 Reading for Writing

One of the reasons to read is to become a better writer. When you read an article that you think is particularly well written, analyze it to see what techniques, words, and phrases seemed to work so well. Reading also expands your knowledge and experience, and can improve your ability to do research. Donald Knuth put it well when he said

> In general when I'm reading a technical paper ... I'm trying to get into the author's mind, trying to figure out what the concept is. The more you learn to read other people's stuff, the more able you are to invent your own in the future.

8 What Next?

Having read the paper you should ask yourself not only what the authors have achieved but also what questions remain. Can you identify open questions that you could answer? Can you see how to combine ideas from this paper with other ideas in a new way? Can you obtain stronger or more general results?